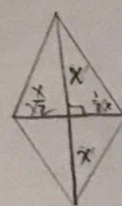
Mock AMC 10 2008

① $5 \to \frac{1}{5}$ $\frac{25}{5} = \boxed{5}$

②



$\left(\frac{1}{2}(2x \cdot x)\right)4 = 2(2x^2)$

$\left(\frac{1}{2}(x \cdot \frac{1}{2}x)\right)4 = 2\left(\frac{x^2}{2}\right) = \boxed{x^2}$

③ $(x + k = 14.20) - 4$

$4x + k = 37.30$

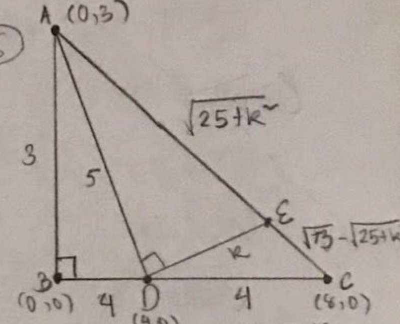
$-3k = -19.50$

$\boxed{k = 6.50}$

1420
4
-56.80
37.30
-19.50

$\frac{6.50}{3)19.50}$ 18, 15

⑥



$3^2 + (2x)^2 = (\sqrt{73})^2$

$9 + 4x^2 = 73$

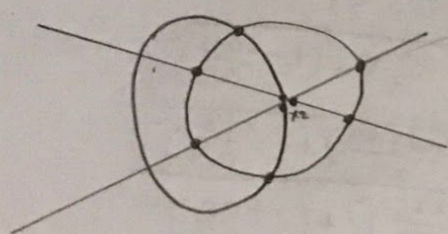
$4x^2 = 64$

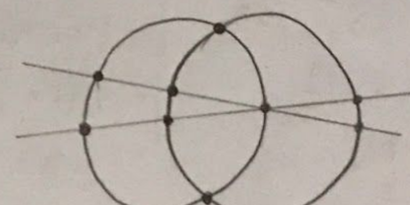
$x^2 = 16$

$x = 4$

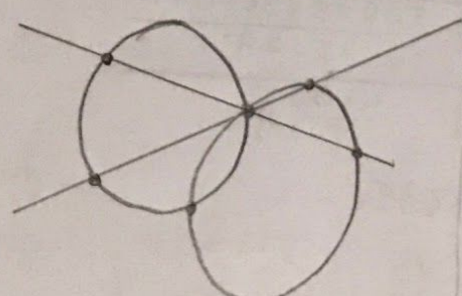
(0,3) $\frac{0-3}{4-0} = -\frac{3}{4}x + 3 = y$
(4,0)

④

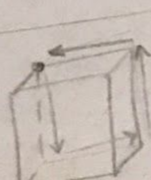


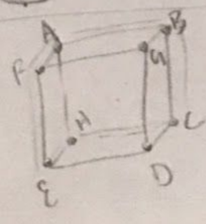


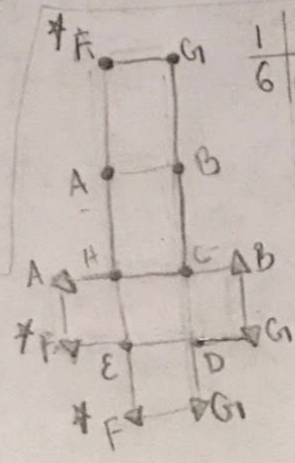
$\boxed{9}$ ↑



⑤







1	2	3	4	5	6
6	6	8	7	$\boxed{9}$	

(6) $y = -\frac{3}{4}x + 3$

$y_\perp = \frac{4}{3}x + k$

$\frac{4}{3}(4) + k = 0$

$\frac{16}{3} + k = 0$

$\therefore k = -\frac{16}{3}$

$y_\perp = \frac{4}{3}x - \frac{16}{3}$

$y = \frac{4}{3}\left(\frac{600}{123}\right) - \frac{16}{3}$

$y = \frac{800}{123} - \frac{16}{3}$

$y = \frac{800}{123} - \frac{656}{123}$

$y = \frac{144}{123}$

(0,3) (8,0) $\frac{0-3}{8-0} = m$

$y = \frac{-3}{8}x + 3$

$y = \frac{4}{3}x - \frac{16}{3} = -\frac{3}{8}x + 3$

$\left(\frac{4}{3} + \frac{3}{8}\right)x = \frac{16}{3} + 3$

$\left(\frac{32}{24} + \frac{9}{24}\right)x = \frac{16}{3} + \frac{9}{3}$

$\frac{41}{24}x = \frac{25}{3}$

$123x = 600$

$x = \frac{600}{123}$

$(x_1, y_1) = \left(\frac{600}{123}, \frac{144}{123}\right)$

$(x_2, y_2) = (0, 3)$

$\Delta D = \sqrt{\left(3 - \frac{144}{123}\right)^2 + \left(0 - \frac{600}{123}\right)^2}$

$\Delta D = \sqrt{\left(\frac{369-144}{123}\right)^2 + \left(-\frac{600}{123}\right)^2}$

$\Delta D = \sqrt{\left(\frac{225}{123}\right)^2 + \left(-\frac{600}{123}\right)^2}$

$\Delta D = \sqrt{\frac{50625 + 360000}{15129}}$

$\Delta D = \sqrt{\frac{410625}{15129}}$

$25 < \Delta D < 35$

Side calculations:

41 × 16 = 656; 800 − 656 = 144; 123 × 3 = 369

225 × 225 = 50625; 600 × 600 = 360000; 360000 + 50625 = 410625; 123 × 123 = 15129

15129 × 5 = 75645; 15129 × 15 = 226935; 15129 × 25 = 378225 (1); 15129 × 35 = 529515 (2)

73 × 73 = 5329 (2)

15129 = 3 × 5043; 5043 = 3 × 1681

$\sqrt{410625}$ / $3\sqrt{1681}$

25 × 24 = 100 (side: 2, 25, 24, 100, 0)

$\sqrt{410625}$ / $3\sqrt{1681}$

410625

9 45625

25 1823

136875
3 | 410625
36
11 0
20 15
26 25
22 1
1

45625
9 | 410625
36
50
45
56
54
22
18
4

1520
3 | 45625
3
15
15
0 5
6
0 2
1823
25 | 45625
25
206
200
62
50
125

15 208
15
5
35
150
25
4
7 5

$\frac{\sqrt{9}\sqrt{25}\sqrt{1823}}{3\sqrt{1681}} = \frac{15\sqrt{1823}}{3\sqrt{1681}} = \frac{5\sqrt{1823}}{\sqrt{1681}}$

41
41
41
1640
1681

(8) women = ●
men = ○

5 | ● ○
4.5/2 | ● ●
4.5/2 | ○ ○

$5 + 2\left(\frac{4.5}{2}\right)$

125

(7)

x	A	B	C	O
A	AAA	ABB		
B	BAA	BBB		
C	CAA	CBB		
O	OAA			
x	A	B	C	O

(9)

1 2 4 8 16 32 64 71 — 7

1 2 3 6 12 24 48 — 9

1 2 4 8 16 32 33 34 35 70 71 — 12

1 2 4 8 16 17 34 68 69 70 71

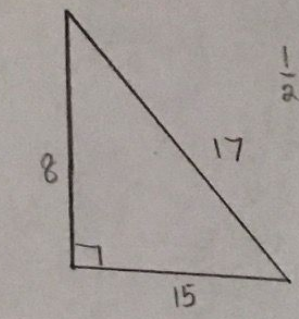
9 1 2 4 8 16 17 34 35 70 71

x	A	B	C	O
A	AA	AB	~~AC~~	~~AO~~
B	BA	BB	~~BC~~	~~BO~~
C	CA	CB	CC	~~CO~~
O	~~OA~~	~~OB~~	~~OC~~	~~OO~~

$\frac{9}{16}$ $\frac{6}{64} = \frac{2}{32} = \frac{1}{16}$

$\frac{3}{64}$

(10)



$\frac{1}{2} \cdot 8 \cdot 15 = \frac{1}{2} \cdot 17 \cdot y$

$8 \cdot 15 = 17 \cdot x$

$\frac{120}{17} = x$

$17 \overline{)120}$ 7

6 5 4
17 17 17
9 8 7
153 136 119

(11) $c = c$

$x = \frac{1}{2}c$

$y = \frac{1}{2}x = \frac{1}{4}c$

$z = \frac{1}{2}y = \frac{1}{8}c$

$c + \frac{1}{2}c + \frac{1}{4}c + \frac{1}{8}c = 105$

$\frac{8}{8}c + \frac{4}{8}c + \frac{2}{8}c + \frac{1}{8}c = 105$

$\frac{15}{18}c = 105$

$15c = 105 \cdot 18$

$c = \frac{105 \cdot 18}{15}$

$c = \frac{105}{3} \cdot \frac{8}{5}$

$c = 35 \cdot \frac{18}{5}$

$c = \frac{18 \cdot 35}{5}$

14
4
50

(12) $2x^3 - 5x^2 + 4x - 10 = 0$

$x^2(2x-5) + 2(2x-5) = -9$

$(x^2+2)(2x-5) = -9$

$x^2+2 = 3$ | $x^2+2 = -3$ | $3 \cdot -3$

$x^2 = 1$ | $x^2 = -5$ | -9

$x = 1$ | $x = \sqrt{-5}$

$(-5+2)(2(\sqrt{-5})-5)$

$x^2+2 = 1$ | $x^2+2 = -9$

$x^2 = -1$ | $x^2 = -11$

$x = \sqrt{-1}$

4
105
18
840
1050
1890

$C = 7 \cdot 18$

$Z = \frac{1}{8}(7 \cdot 18)$

5 18
7
126
8

15
8 126
56
56
0
6

$\frac{1}{8}\left(\frac{105 \cdot 18}{15}\right)$

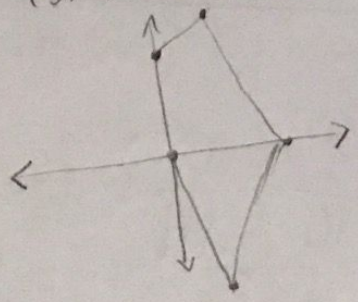
$\left(\frac{1}{8} \cdot \frac{1890}{15}\right)$

$\left(\frac{1}{4} \cdot \frac{945}{15}\right)$

$\left(\frac{1}{4} \cdot \frac{189}{3}\right)$

$\left(\frac{1}{4} \cdot 63\right) \Rightarrow \frac{63}{4} = 14$

(13) $(0,0)(3,4)(6,0)$
$(3,-4)(0,3)$



0 0 0
3 4 0
6 0 24
3 4 0
0 3 0
24
9

$\frac{9}{2} = 4.5$

(14) $xy = x + y = xy$

$10x + y = x + y = xy$

$10x + y = x + xy + y$

$9x = xy$

$9 = y$

$\frac{1260}{9} = 140$

19 49 79 } 9+9 } 81
29 59 89 } 450 } 450
39 69 99 } } 126

19
29
39
49
59
69
79
89
99
531

$\frac{531}{9} = 59$

(15) $1! + 1\cdot 2! + 1\cdot 2\cdot 3! + \dots + 1\cdots 1337!$

$1! + 2!\left(1 + \underbrace{\frac{3!}{2!}\left(1 + \frac{4!}{3!}\left(\dots\right)\right)}_{\text{even}}\right)$

$6! = 720$
$7! = 5040$
$8! = 40320$
$9! = 362880$

$1! = 1$ $1^{\sim} = 1$
$2! = 2$ $1^{\sim} + 2^{\sim} = 1 + 4 = 1 + 16 = 17$
$3! = 6$ $1^{\sim} + 2^{\sim} + 3^{\sim} = 1 + 2 + 6 = 1 + 16 + 36$
$4! = 24$ $1^{\sim} + 2^{\sim} + 3^{\sim} + 4^{\sim} = 1 + 2 + 6 + 24 =$
$5! = 120$

$(1!)^{\sim} + (2!)^{\sim} + \dots + (1337!)^{\sim} \pmod 5$

$1! + 2! + \dots + (1337!) \pmod 5$

$\underbrace{1+2}_{3} + \underbrace{6 + 24}_{30} + 5(\sim)$

$(1!)^{\sim} + (2!)^{\sim} + \dots + (1337!)^{\sim} \pmod 9$

$1^{\sim} + 2^{\sim} + \dots + (\sim)^{\sim} \pmod 9$

$1^{\sim} + 720 + 5040 + 40320 + 362880$

$1 + 2 + 6 + 24 + 120 + 720 + 5040 + 40320$

$873 \pmod{770}$

$46233 \pmod 9 \equiv 18$

(16)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66
1 13 78 286 715 1287 1716 1716 1287 715 286
1 14 91 364 1001 2002 3003 3432 3003

(17) $2(lw+ln+wn)=48$

$3\cdot4+3x+4x=48$

$12+7x=46$

$7x=36$

$x=\frac{36}{7}$

$\frac{36}{7}\cdot3\cdot4=\frac{12\cdot36}{7}$

(18) $\binom{7}{4}=\frac{7!}{3!}$

$\binom{7}{4}=\frac{7\cdot6\cdot5\cdot4}{42\ \ 20}$

840

ABCD

ABC DBC
ACB DCB
BAC DAC
BCA DCA
CBA DBA
CAB DAB

BDC BCD
CDB CBD
ADC ACD
CDA BAD
BDA ABD
ADB

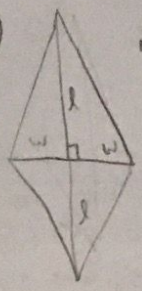
$\binom{4}{3}=24$

$=\frac{n!k!}{k!(n-k)!}$

$=\frac{4!3!}{3!(4-3)!}$

$\frac{4\cdot3\cdot2\cdot1}{3\cdot2\cdot1(1)!}=4$

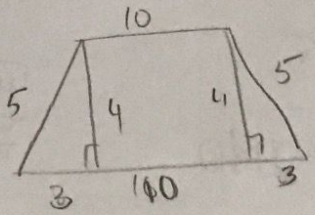
$\frac{4\cdot3\cdot2\cdot3\cdot2}{3\cdot2}$

(19) 

$4\left(\frac{1}{2}\cdot lw\right)$

$2lw=24$

$lw=12$

(20) 

$\left(\frac{1}{2}\cdot3\cdot4\right)2+40$

56

$\frac{56}{\pi}=\frac{4(\pi r^2)}{16}\Rightarrow56\cdot16=4(\pi^2r^2)$

$\sqrt{56\cdot4}=\sqrt{\pi^2r^2}$

$2\sqrt{56}=\pi r$

$\frac{56}{\pi}=\frac{16}{4\pi r^2}$

$56\pi r^2=16\pi$

$16\pi r^2=56\pi$

πr^2

$16r^2=56$

$r^2=\frac{56}{16}=\frac{14}{4}$

$56\cdot4r^2=16$

$16r^2=56\cdot4$

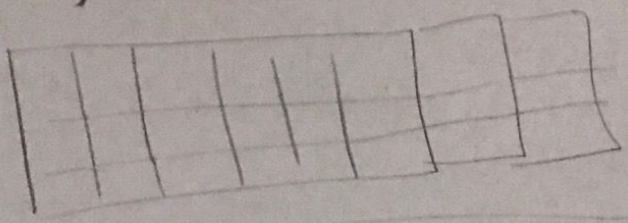
$56r^2=4$

$56r^2=16$

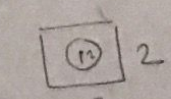
$r^2=\frac{4}{56}=$

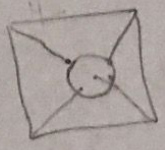
$\frac{56}{4}=14$

(21) $\binom{24}{12} = \frac{n!}{(n-k)!} = \frac{24!}{12!}$??



(22)





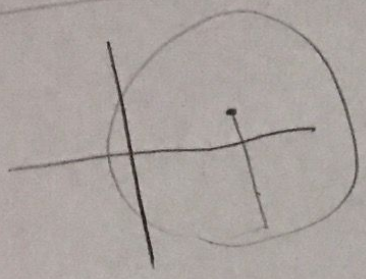
$2+\sqrt{2}$

$r = \frac{\sqrt{2}}{2}$

$\left(4 - \frac{\sqrt{2}}{2}\right) \cdot \frac{2}{2}$

$\frac{8-\sqrt{2}}{8}$

(23)



$100 = 2^{\square} 5^{\square}$

$2^5 \quad 4 \quad (3)(2)(2) = 9$

$5^5 \quad 2^2$

$\pi\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\pi}{2}$

$\frac{4 - \frac{\pi}{2}}{4} \cdot \frac{2}{2}$

$\frac{8-\pi}{8}$